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SOME INVARIANT RELATIONS IN THE PROBLEM OF THE MOTION OF A BODY ON A SMOOTH HORIZONTAL PLANE*

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For the motion of a heavy rigid body on a smooth horizontal plane, we find in parametric form general analytical expression for the bounding surface of the body, such that the equations of motion admit of four linear and velocity-homogeneous invariant relations. We consider in detail the special case when the body is bounded by a surface of revolution whose axis of symmetry is the axis of one of the circular sections of the central gyration ellipsoid. The equations of motion of this body admit of a single invariant relation. We determine the corresponding stationary motions of the body and derive a sufficient condition of stability of some manifolds of stationary motions.

1. Consider the system of differential equations

$$dx/dt = X(x, t),\ x \in R^l,\ t \in R \tag{1.1}$$

defined in some domain of variables.

The manifold Σ is called invariant if it consists of integral curves of system (1.1). In the neighbourhood of an arbitrary interior point $x_0 \in \Sigma$, the $(l-k)$-invariant manifold is defined by the equations

$$F_1(x, t) = 0, \ldots, F_k(x, t) = 0 \tag{1.2}$$

where the equalities (1.2) constitute the set of invariant relations of system (1.1). These relations may include first integrals of the system (the constants of integration are incorporated in the functions F_i), single invariant relations each locally defining a $(l-1)$-dimensional invariant manifold.

Eqs.(1.2) carry some information on the behaviour of the solutions of system (1.1) and sometimes make it possible to identify important classes of particular solutions of the system by elementary techniques /1/.

For the equations of mechanics, the study of invariant manifolds (their existence and their properties) should naturally start with the simple case when the corresponding functions F_i are linear in the generalized velocities $q^{\cdot} \in R^n$,

$$F_1 = \sum_{i=1}^{n} c_{1i} q^{i\cdot} = 0, \ldots, F_k = \sum_{i=1}^{k} c_{ki} q^{i\cdot} = 0 \quad (k < n) \tag{1.3}$$

($c_{ij} = c_{ij}(q)$, $q \in R^n$ is the vector of generalized coordinates of the mechanical system). The conditions for some such manifolds to exist were derived in /2/.

Below we use the following result /2/. Let

$$T = \frac{1}{2} \sum_{i,j=1}^{n} a_{ij}(q) q^{i\cdot} q^{j\cdot}, \quad U = U(q)$$

be respectively the kinetic energy and the force function of a holonomic system with n degrees of freedom. The Lagrange equations admit of $n-1$ invariant relations of the form (1.3) if, and for $U \not\equiv \mathrm{const}$ only if, the first differential parameter is a function of U:

$$\Delta_1 U = \sum_{i,j=1}^{n} a^{ij}(q) \frac{\partial U}{\partial q^i} \frac{\partial U}{\partial q^j} = \alpha(U) \tag{1.4}$$

If condition (1.4) holds, then the force lines are geodesics of the configuration manifold of the system /3/.

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2. Let us apply this criterion to the following problem. A heavy rigid body bounded by a regular convex surface moves on an absolutely smooth horizontal plane. Let $Ox_1x_2x_3$ be the Cartesian system of coordinates formed by the principal central axes of inertia of the body, $(\gamma_1, \gamma_2, \gamma_3)$ and $(\omega_1, \omega_2, \omega_3)$ the components in these axes of the unit vector along the vertical and the instantaneous angular velocity of the body respectively, (x, y, z) the coordinates of the point O in the Cartesian system whose xy plane coincides with the supporting plane, A, B, C the principal central moments of inertia, and M the mass of the body.

In general

$$z = \beta(\gamma_1, \gamma_2, \gamma_3) \tag{2.1}$$

where the function β is determined by the shape of the bounding surface and by the mass distribution in the body.

The kinetic energy (multiplied by 2) and the force function are given by (P is the weight of the body)

$$\begin{aligned} 2T &= M(x^{\cdot 2} + y^{\cdot 2}) + (A + Mu_1^2)\omega_1^2 + (B + Mu_2^2)\omega_2^2 + \\ &\quad (C + Mu_3^2)\omega_3^2 + 2M(u_1u_2\omega_1\omega_2 + u_2u_3\omega_2\omega_3 + u_3u_1\omega_3\omega_1) \\ u_1 &= \beta_2\gamma_3 - \beta_3\gamma_2, \ \beta_1 = \partial\beta/\partial\gamma_1 \ (123) \\ U &= -P\beta \end{aligned} \tag{2.2}$$

Omitting the case $U \equiv \text{const}$ (a symmetric sphere), we will derive the conditions when the equations of motion of the body admit of a set of four linear invariant relations that are homogeneous in $\omega_1, \omega_2, \omega_3$.

Since the equations of motion have three cyclic integrals

$$Mx^{\cdot} = c_1, \ My^{\cdot} = c_2, \ A\gamma_1\omega_1 + B\gamma_2\omega_2 + C\gamma_3\omega_3 = c_3$$

(c_1, c_2, c_3 are arbitrary constants), thus required conditions guarantee the existence of an additional linear relation, which forms an invariant set together with these integrals for $c_1 = c_2 = c_3 = 0$.

The invariant $\Delta_1\beta$ is conveniently evaluated in the quasicoordinates π_1, π_2, π_3 corresponding to the quasivelocities $\omega_1, \omega_2, \omega_3$. Here

$$\frac{\partial}{\partial\pi_i} = \gamma_{i-2}\frac{\partial}{\partial\gamma_{i-1}} - \gamma_{i+1}\frac{\partial}{\partial\gamma_{i+2}}$$

(the subscripts are written mod 3).

We have

$$\Delta_1\beta = \sum_{i,j=1}^{3}\left(\frac{\partial^2 T}{\partial\omega_i\partial\omega_j}\right)^{-1}\frac{\partial\beta}{\partial\pi_i}\frac{\partial\beta}{\partial\pi_j} = \frac{1}{M} - \frac{1}{M + M^2\Lambda}$$

$$(\Lambda = au_1^2 + bu_2^2 + cu_3^2, \ a = A^{-1}, \ b = B^{-1}, \ c = C^{-1})$$

Therefore, by the definition of the force function (2.2), condition (1.4) reduces to the form $\Lambda = \alpha_1(\beta)$ $(\alpha_1 > 0)$. This equality may be rewritten as

$$\begin{aligned} &a\left(\gamma_2\frac{\partial\sigma}{\partial\gamma_3} - \gamma_3\frac{\partial\sigma}{\partial\gamma_2}\right)^2 + b\left(\gamma_1\frac{\partial\sigma}{\partial\gamma_3} - \gamma_3\frac{\partial\sigma}{\partial\gamma_1}\right)^2 + \\ &\quad c\left(\gamma_2\frac{\partial\sigma}{\partial\gamma_1} - \gamma_1\frac{\partial\sigma}{\partial\gamma_2}\right)^2 = 1 \end{aligned} \tag{2.3}$$

$$\sigma = \int\frac{d\beta}{\sqrt{\alpha_1(\beta)}}, \quad \beta = f(\sigma) \tag{2.4}$$

Treating σ as an unknown function, we note that (2.3) is the Hamilton-Jacobi equation in the problem of inertial motion of a rigid body around a fixed point with the supplementary constraint that the projection of the angular momentum vector of the body on the vertical vanishes (and the constant in the energy integral is $h = {}^1/_2$). This equation is integrated in generalized coordinates /4/.

Let us consider tow cases: 1) some of the numbers A, B, C are equal, 2) $A > B > C$.

In the first case, denoting by θ the angle between the axis of symmetry of the body and the vertical, we may rewrite Eq.(2.3) in terms of Euler angles φ, ψ, θ. The coordinates φ, ψ are cyclic. Therefore, by (2.4), we find that the height of the centre of mass of the body above the supporting plane is an arbitrary function of the angle θ. Only solids of revolution have this property. The equations of motion of a solid of revolution are integrable in quadratures /5/.

In the second case, we make the substitution

$$\gamma_1^2 = \frac{(a+\lambda)(a+\mu)}{(a-b)(a-c)}, \quad \gamma_2^2 = \frac{(b+\lambda)(b+\mu)}{(b-c)(b-a)} \tag{2.5}$$

$$(-c < \mu < -b,\ -b < \lambda < -a)$$

in the equation

$$\left[a\left(\frac{\partial V}{\partial\gamma_2}\right)^2 + b\left(\frac{\partial V}{\partial\gamma_1}\right)^2\right](1-\gamma_1^2-\gamma_2^2) + c\left(\gamma_2\frac{\partial V}{\partial\gamma_1} - \gamma_1\frac{\partial V}{\partial\gamma_2}\right)^2 = 1 \tag{2.6}$$

$$V(\gamma_1, \gamma_2) = \sigma(\gamma_1, \gamma_2, \sqrt{1-\gamma_1^2-\gamma_2^2}) \tag{2.7}$$

which is the restriction of Eq.(2.3) on the submanifold $\gamma_3 - \sqrt{1-\gamma_1^2-\gamma_2^2} = 0$. Here λ, μ are elliptical coordinates on the surface of the central ellipsoid of inertia of the body

$$x_1^2/a^2 + x_2^2/b^2 + x_3^2/c^2 = 1 \tag{2.8}$$

In the variables λ, μ, Eq.(2.6) takes the form

$$\frac{\varkappa(\lambda)}{\lambda}\left(\frac{\partial V}{\partial\lambda}\right)^2 - \frac{\varkappa(\mu)}{\mu}\left(\frac{\partial V}{\partial\mu}\right)^2 = \frac{1}{\mu} - \frac{1}{\lambda} \tag{2.9}$$

$$(\varkappa(x) = (a+x)(b+x)(c+x))$$

i.e., the variables are separated. The complete integral of this equation is (e_1, e_2 are constants)

$$V = r(-b, e_1, \lambda) + r(-c, e_1, \mu) + e_2 \tag{2.10}$$

$$\left(r(s, k, \rho) = \int_s^\rho \sqrt{(kx-1)\varkappa^{-1}(x)}\,dx\right)$$

By the theory of /6/, in order to obtain a general solution of Eq.(2.9), we have to assign constant values to the parameters e_1, e_2 in (2.10) or alternatively to link them by a single dependence, e.g., $e_2 = f(e_1)$, and to substitute into (2.10) the solution $e_1(\lambda, \mu)$ of the integral equation

$$r_1(-b, e_1, \lambda) + r_1(-c, e_1, \mu) + df_1/de_1 = 0$$

$$\left(r_1(s, k, \rho) = \frac{1}{2}\int_s^\rho \frac{x\,dx}{\sqrt{(kx-1)\varkappa(x)}}\right)$$

Formulas (2.1), (2.4), (2.7) and (2.10) can be used to express the height z of the centre of mass of the body above the supporting plane as a function of the variables λ, μ. The expression for z includes two arbitrary functions f and f_1 constrained by the single condition that the surface S of the body is convex.

Let us derive a parametric equation of the surface S. For an arbitrary position of the body, the vertical through the centre of mass O passes through the surface (2.8) at the point with the values λ_0, μ_0 of the elliptical coordinates λ, μ. If these values λ_0, μ_0 of the Gaussian coordinates are associated with the point of tangency O_1 of the surface S with the supporting plane, then the parametric form of the surface S is globally defined, since the surface is convex.

We have

$$z(\lambda, \mu) = l\gamma_1 + m\gamma_2 + n\gamma_3 \tag{2.11}$$

$$\gamma_3^2 = 1 - \gamma_1^2 - \gamma_2^2 = \frac{(c+\lambda)(c+\mu)}{(c-a)(c-b)}$$

where (l, m, n) are the components of the radius-vector OO_1 in the axes $Ox_1x_2x_3$, and γ_1, γ_2 are determined from the formulas (2.5).

Differentiating (2.11), we find

$$\partial_\rho z = l\partial_\rho\gamma_1 + m\partial_\rho\gamma_2 + n\partial_\rho\gamma_3 \quad (\partial_\rho = \partial/\partial\rho,\ \rho = \lambda, \mu)$$

Hence, using (2.11), we obtain

$$l = \frac{\gamma_1}{\lambda-\mu}\left[(\lambda-\mu)z - 2(b+\lambda)(c+\lambda)\partial_\lambda z + 2(b+\mu)(c+\mu)\partial_\mu z\right] \tag{2.12}$$

$$(lmn,\ 123,\ abc)$$

These formulas give the required parametric description of the bounding surface of the body.

For bodies whose surfaces belong to this class and only for such bodies (the case of a symmetric sphere is not considered), the equations of motion admit of a set of four invariant relations of the form (1.3). For instance, this set may comprise the following relations:

$$x^{\cdot} = 0,\ y^{\cdot} = 0,\ A\gamma_1\omega_1 + B\gamma_2\omega_2 + C\gamma_3\omega_3 = 0,\ Au_3\omega_1 - Cu_1\omega_3 = 0$$

They are linearly independent and their coefficients satisfy the orthogonality condition.

3. Let us consider in more detail the special case when e_1 and e_2 are constant.

The functions $r(-b, e_1, \lambda)$ and $r(-c, e_1, \mu)$ in (2.10) take real values only if we respectively have

$$e_1 \geqslant 1/\lambda \quad \text{for} \quad -b < \lambda < -a$$
$$e_1 \leqslant 1/\mu \quad \text{for} \quad -c < \mu < -b$$

Therefore, the parameter e_1 may only have a unique constant value

$$e_1 = -1/b = -B \tag{3.1}$$

For this value of e_1, the integrals in (2.10) are expressible in terms of elementary functions.

Let us determine the geometry of the body. Let H be the foot of the perpendicular dropped from the centre of mass O to the supporting plane, and N the point where the axis of one of the circular sections of the central gyration ellipsoid meets the supporting plane. The vector with the components $(\pm\sqrt{b-a}, 0, \sqrt{c-b})$ in the axes $Ox_1x_2x_3$ points in the direction ON. Using Eqs.(2.5) for the components of the vertical vector and the parametric Eqs.(2.12) of the surface of the body for the case (3.1), we can show that for any λ, μ the points O, O_1, H, N lie in the same plane p.

Let us determine the position of the vector of the kinematically feasible angular velocity ω relative to the body if the plane p is fixed in stationary space and yet another constraint is imposed, $OH = \text{const}$. Since the instantaneous velocity of the point O_1 of the body lies in the supporting plane, the vector ω belongs to the p plane (we assume that O_1 and H are two distinct points). Since the ON axis also belongs to p, we conclude that the vectors ON and ω are collinear. Thus, the body may only rotate around a fixed axis ON. Since OH is arbitrary, we conclude that in case (3.1) the body is bounded by a surface of revolution whose axis of symmetry is the axis of one of the circular sections of the central gyration ellipsoid of the body.

The equations of motion of this body have a single invariant relation /7/

$$c\sqrt{b-a}\,\omega_1 \pm a\sqrt{c-b}\,\omega_3 = 0 \tag{3.2}$$

In a system with known invariant relations (in particular, with known first integrals), stationary points of one invariant relation given the values of the constants in the other relations correspond to so-called stationary motions of the system /1, 8/.

Let us find the stationary motions with respect to the kinetic energy integral of a body with a surface of revolution whose axis of symmetry passes through the centre of mass of the body, lies in the plane orthogonal to the mid-axis of the central inertia ellipsoid, and makes an acute angle θ with the smallest axis of this ellipsoid,

$$\operatorname{tg}\theta = \mp\sqrt{(c-b)/(b-a)} \tag{3.3}$$

(the corresponding signs should be used in Eqs.(3.2) and (3.3)). Without loss of generality, we can also assume that the projection of the centre of mass of the body on the supporting plane remains fixed, i.e., $x^{\cdot} = y^{\cdot} = 0$.

It is helpful to replace the variables γ_i with three angles: φ — the angle of natural rotation of the body about the axis of symmetry, ψ — the turning angle of the vertical plane of the meridional section of the body, and α — the angle of inclination of the axis of symmetry to the horizon $(-\pi/2 \leqslant \alpha \leqslant \pi/2)$.

The relationship between $\omega_1, \omega_2, \omega_3$ and $\psi^{\cdot}, \alpha^{\cdot}, \varphi^{\cdot}$ for $\cos\alpha \neq 0$ is given by

$$\omega_1 = \psi^{\cdot}(\sin\alpha\cos\theta - \cos\alpha\sin\theta\cos\varphi) - \alpha^{\cdot}\sin\theta\sin\varphi + \varphi^{\cdot}\cos\theta \tag{3.4}$$
$$\omega_2 = -\psi^{\cdot}\cos\alpha\sin\varphi + \alpha^{\cdot}\cos\varphi$$
$$\omega_3 = -\psi^{\cdot}(\sin\alpha\sin\theta + \cos\alpha\cos\theta\cos\varphi) - \alpha^{\cdot}\cos\theta\sin\varphi - \varphi^{\cdot}\sin\theta$$

From (3.2) we obtain (ε is a new variable)

$$\omega_1 = \varepsilon BA^{-1}\sin\theta,\ \omega_3 = \varepsilon BC^{-1}\cos\theta \tag{3.5}$$

Using formulas (3.3)-(3.5), we rewrite the expression for the total mechanical energy of the body in the form

$$H = {}^1/_2(A\omega_1^2 + B\omega_2^2 + C\omega_3^2) + {}^1/_2Mz'^2\alpha^{\cdot 2} + Pz =$$
$${}^1/_2B(\varepsilon^2 + \omega_2^2) + {}^1/_2Mz'^2\Phi'^2(\varphi) + Pz$$
$$\Phi(\varphi) = \varepsilon\cos\varphi + \omega_2\sin\varphi,\ z' = dz/d\alpha,\ \Phi' = d\Phi/d\varphi$$

We have to find the stationary points of this function on the manifold defined by the area integral (given the constant c_3)

$$-B\Phi(\varphi)\cos\alpha = c_3 \tag{3.6}$$

The stationarity conditions are (σ is the Lagrange multiplier)

$$\begin{aligned} &B\varepsilon - Mz'^2\Phi'(\varphi)\sin\varphi - B\sigma\cos\varphi\cos\alpha = 0 \\ &B\omega_2 + Mz'^2\Phi'(\varphi)\cos\varphi - B\sigma\sin\varphi\cos\alpha = 0 \\ &\Phi'(\varphi)\,[Mz'^2\Phi(\varphi) + B\sigma\cos\alpha] = 0 \\ &Mz'd^2z/d\alpha^2\Phi'^2(\varphi) + Pz' + B\sigma\Phi(\varphi)\sin\alpha = 0 \end{aligned} \tag{3.7}$$

From the first two equations of (3.7), we obtain

$$\Phi'(\varphi) = 0 \tag{3.8}$$

Hence,

$$\varepsilon = \sigma\cos\alpha\cos\varphi,\ \omega_2 = \sigma\cos\alpha\sin\varphi \tag{3.9}$$

The third equality in (3.7) is identically true, and the fourth takes the form

$$Pz' + B\sigma\Phi(\varphi)\sin\alpha = 0 \tag{3.10}$$

By (3.4) and (3.5), condition (3.8) implies that $\alpha^{\cdot} = 0$, i.e., in stationary motion the axis of symmetry of the body maintains a constant inclination to the horizon, $\alpha = \alpha_0$.

Let $z'(\alpha_0) \neq 0$.

Formulas (3.9) give the solution of Eqs.(3.7). Here $\sigma = -bc_3\sec\alpha_0$ and the value of α_0 is determined from the equation

$$Pz'(\alpha_0) + bc_3^2\,\mathrm{tg}\,\alpha_0 = 0$$

From formulas (3.4) and (3.5), we obtain

$$\begin{aligned} &\psi^{\cdot} = -\sigma,\ \varphi^{\cdot} = (\mathrm{tg}\,\alpha_0 - k\cos\varphi)\,\sigma\cos\alpha_0 \\ &k^2 = (1 - B/A)\,(B/C - 1),\ \mathrm{sign}\,k = \mathrm{sign}\,\theta \end{aligned} \tag{3.11}$$

The body rotates uniformly around the vertical. If $|\mathrm{tg}\,\alpha_0| > k$, then it also rotates around the axis of symmetry with angular velocity of periodically varying magnitude. If $|\mathrm{tg}\,\alpha_0| \leqslant k$, then the angular velocity of the natural motion of the body decreases monotonically to zero as $t \to \infty$, and the body does not complete a single rotation around the axis of symmetry.

These stationary motions form a 3-dimensional manifold Σ_3 and contain no equilibria. These motions exist only if

$$z'\sin\alpha < 0 \tag{3.12}$$

This inequality holds, for instance, in the neighbourhood of a statically stable equilibrium of an asymmetric sphere. Conversely, for a symmetrical body bounded by an ellipsoidal surface of revolution whose axis coincides with the largest axis of the ellipsoid, inequality (3.15) in general does not hold.

Let $z'(\alpha_0) = 0$.

If $\alpha_0 \neq 0$, then from Eqs.(3.5), (3.9) and (3.10) it follows that every stationary motion of the body is an equilibrium.

For the case $\alpha_0 = 0$, Eqs.(3.11) hold. The stationary motions form a 2-dimensional manifold Σ_2 and include equilibria. For non-equilibrium stationary motions, the smallest and the largest axes of the central inertia ellipsoid of the body tend asymptotically in time to a horizontal attitude.

If the axis of symmetry is vertical, then the stationary motions are realizable equilibria, since by symmetry of the body surface $z'(\pm\pi/2) = 0$.

In order to investigate the stability of stationary motions, consider the function

$$\begin{aligned} R = {} & H + \sigma(f_3 + c_3) + {}^1/_2\sigma^2 B\cos^2\alpha_0 - Pz(\alpha_0) = \\ & {}^1/_2 B(\Omega_1^2 + \Omega_2^2) + {}^1/_2 Mz'^2(\Omega_1\sin\varphi - \Omega_2\cos\varphi)^2 + P[z(\alpha) - \\ & - z(\alpha_0)] + {}^1/_2 bc_3^2(1 - \cos^2\alpha/\cos^2\alpha_0) \\ f_3 = {} & -B\Phi(\varphi)\cos\alpha - c_3,\ \Omega_1 = \varepsilon - \sigma\cos\alpha\cos\varphi,\ \Omega_2 = \\ & \omega_2 - \sigma\cos\alpha\sin\varphi \end{aligned}$$

Let $z(\alpha_0)$ be a strict minimum of the function $z(\alpha)$. Then the function R is positive definite in the variables $\Omega_1, \Omega_2, \alpha - \alpha_0$. Hence /9/, the corresponding manifolds of equilibrium stationary motions and the manifold Σ_3 are conditionally stable relative to the deviations characterized by these variables. Conditional stability is understood in the sense that the initial points of the perturbed motion paths should satisfy Eq.(3.2) and, as we have noted above, the equations $x^{\cdot} = y^{\cdot} = 0$.

Let $z'(\alpha_0) \neq 0$. The condition of positive definiteness of the quadratic form $\delta^2 R$ on the linear manifold $\delta f_3 = 0$:

$$(B \cos \varphi \cos \alpha_0)\, \delta\Omega_1 + (B \sin \varphi \cos \alpha_0)\, \delta\Omega_2 + 2c_3 \sin \alpha_0\, \delta\alpha = 0$$

leads to the single inequality

$$b^2c_3^4 + 3\,[Pz'(\alpha_0)]^2 + bc_3^2Pz''(\alpha_0) > 0$$

Thus, for sufficiently large values of the angular velocity, the manifold Σ_3 of the stationary motions of the body is conditionally stable relative to these deviations.

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SHOCK INTERACTION BETWEEN A CONCENTRATED OBJECT AND A ONE-DIMENSIONAL ELASTIC SYSTEM*

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A physical interpretation of the results obtained earlier**(**Malanov S.B. and Utkin G.A. Formulation of a problem of shock interaction between a concentrated object and a one-dimensional elastic system. Gor'kii, 1986. Dep. at VINITI 5.12.86, 8304-B86.) for the shock interaction of a homogeneous elastic system with a concentrated object is given in the form of the laws of variation of the energy and moments. The impact of a material point against a string is considered as an example, and the dependence of the time of contact and the coefficient of restitution on the parameters of the problem is given.

The problem of the correct conditions at the point of contact and of relations holding at the beginning and end of contact were solved in /1, 2/, where additional geometrical and physical concepts (laws of conservation of energy, momentum, etc.) were brought in. The study of a

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